日仏力学系研究集会

L'atelier Franco-Japonais

sur la dynamique réelle et complexe des applications de Hénon

下記の要領で研究集会を開催します。皆様のご参加をお待ちしています。

日時： ３月２７日（月）9:00 から３１日（金）17:15 まで

会場： 京都大学理学部数学教室 3-110 号室

主催者： Pierre Berger (Sorbonne), Yutaka Ishii (Kyushu)

プログラム：

３月２７日（月）

9:00 – 9:45	Shigehiro Ushiki (Kyoto)
10:00 – 10:45	Shin Kiriki (Tokai)
11:00 – 11:45	Sébastien Biebler (Paris Cité)
13:30 – 16:30	discussion
16:30 – 17:15	Hiroki Takahasi (Keio)

３月２８日（火）

9:00 – 9:45	Raphaël Krikorian (École Polytechnique)
10:00 – 10:45	Mao Shinoda (Ochanomizu)
11:00 – 11:45	Takumi Yagi (Kyoto)
13:30 – 16:30	discussion
16:30 – 17:15	Ziyuan Zhang (Paris Nord)

３月２９日（水）

9:00 – 9:45	Reimi Irokawa (NTT)
10:00 – 10:45	Bernhard Reinke (Sorbonne)
11:00 – 11:45	Thomas Richards (Warwick/Kyushu)
	free afternoon

３月３０日（木）

9:00 – 9:45	Sylvain Crovisier (Paris Saclay)
10:00 – 10:45	Yuki Hironaka (Kyushu)
11:00 – 11:45	Mathieu Helfter (Sorbonne)
13:30 – 16:30	discussion
16:30 – 17:15	Zin Arai (Chubu)
18:00 –	banquet

３月３１日（金）

9:00 – 9:45	no talk
10:00 – 10:45	Hiroyuki Inou (Kyoto)
11:00 – 11:45	Yohan Taflin (Bourgogne)
13:30 – 17:15	discussion

タイトルとアブストラクト：

Shigehiro Ushiki (Kyoto)

Exotic rotation domains in complex Hénon dynamics

Fatou component of complex dynamical system is called a rotation domain if the dynmics in the set is quasiperiodic. The closure of the orbit of almost any initial point is a circle or a torus. We say a rotation domain is exotic if the domain is not simply connected. In this talk, we explain how to observe such object numerically.

Shin Kiriki (Tokai)

$C^{1+\alpha}$-robust tangencies and pluripotent wandering domains for surface diffeomorphisms

In this study we introduce the property of pluripotency. This means that arbitrarily small perturbations can stochastically realize arbitrary dynamics, including arbitrary recurrent dynamics or combinations thereof, arbitrary Dirac physical measures as well as historic behavior. In other words, it can be thought of as a stochastic version of universal dynamics. The pluripotent property has been suggested that this property also exists in 2-dimensional C^2 Newhouse domains [Colli-Vargas '01, Kiriki-Soma '17] and in the 3-dimensional C^1 domains [arXiv 2107.09844]. In this study we first prove the existence of a 2-dimensional $C^{1+\alpha}$ Newhouse domain $\mathcal{O}$, which is folklore, and besides show the existence of a persistent class of $\mathcal{O}$ whose every element has a pluripotent wandering domain.

Hiroki Takahasi (Keio)

On the LGYK conjecture on bifurcations of the real Henon family with small Jacobian

Lai, Grebogi, Yorke, Kan [Nonlinearity, 1993] performed a crude numerical experiment on the dynamics near the bifurcation parameter of the real Henon family from Smale's horseshoe, and conjectured that "most" parameters from the bifurcation before the creation of the global chaotic attractor correspond to a non-hyperbolic chaotic transient (in particular, no attractor). I will roughly explain how to prove a weaker version of this conjecture asserting that "most" parameters correspond to a chaotic transient. To my knowledge, the hyperbolicity or non-hyperbolicity of the transient remains essentially open.

Mao Shinoda (Ochanomizu)

Ergodic optimization and rotation sets for symbolic dynamical systems

Ergodic optimization is the study of maximizing measures which are invariant measures with the maximum ergodic average for a given potential and has attracted recent research interest. Garibaldi and Lopes introduced rotation numbers as constraints in Ergodic optimization and prove fundamental properties in this context such as generic uniqueness. In this talk we consider genreric zero entropy property in this constrained case, which is also an analogous problem with a unconstrained Ergodic optimization.

Takumi Yagi (Kyoto)

An approach to non-radial perturbations of semi-parabolic Hénon maps

We consider the Hénon maps given by $H(x, y) = (x^2 + c + ay, ax), (c, a) \in \mathbb{C}^2$, where $|a|$ is sufficiently small. Hénon maps with semi-parabolic fixed points are not hyperbolic, difficult to deal with dynamically, and the Julia sets nontrivially vary on a neighborhood of a semi-parabolic parameter. On perturbations $\{H_t\}_{t \in [0, \epsilon]}$ of such Hénon maps, we see bifurcation phenomena where semi-parabolic fixed points bifurcate into fixed points and periodic points. We see that if the eigenvalues $\lambda_t, t \in [0, \epsilon]$ at bifurcated fixed points, satisfying that $\lambda_0^m = 1$ for some $m \in \mathbb{N}$ and $|\lambda_t| \neq 1$ for $t \neq 0$, have some nice condition, then the Julia sets vary continuously. Radu and Tanase showed if $\lambda_t = (1 \pm t)\lambda_0$, then the Julia sets vary continuously and $H_t, t \neq 0$ are hyperbolic. Let $\lambda_t/\lambda_0 = \exp(L_t + i\theta_t)$ and $\theta_t \to 0$ as $t \to 0$. We say that $\{H_t\}$ is a *radial perturbation* if $\theta_t = O(L_t)$. The results above hold if $\theta_t = O(L_t)$. In this talk, we introduce an approach to non-radial perturbations.

Reimi Irokawa (NTT)

Non-archimedean and hybrid dynamics of Hénon maps

To study of the meromorphic degeneration of dynamics, the theory of hybrid spaces, established byFavre, is known to be a strong tool. In this talk, we apply this theory to the dynamics of Hénon maps; for a famiy of Hénon maps $\{H_t\}_t$ parametrized by a unit punctured disk meromorphically degenerating at the origin, we show that as $t \to 0$, the family of the invariant measures $\{\mu_t\}$ "weakly converges" to the measure on the Berkovich affine plane which is naturally defined by the family $\{H_t\}_t$, in the sense of the theory of hybrid spaces.

Thomas Richards (Warwick/Kyushu)

Monodromy and complex Hénon maps

In 1991, Blanchard, Devaney, and Keen proved that loops in the shift locus of degree d polynomials induce all automorphisms of the one-sided shift

on d symbols. Hubbard then conjectured that an analogous result holds for complex Hénon maps. His student, Chris Lipa, investigated this conjecture experimentally, proposing a relationship between loops around certain empirically observed components of the horseshoe locus and the combinatorics of the Mandelbrot set.

In this talk, we will discuss the conjectures and describe a new characterisation of the relevant parameter space components. We will also describe some experimental work aiming to understand the mechanism of monodromy in Hénon parameter space. Both of these utilise a new concept of critical point for Hénon maps which mimics the behaviour we see in the one-dimensional monodromy problem.

Yuki Hironaka (Kyushu)

Symbolic dynamics for Hénon maps near the boundary of the horseshoe locus

In this talk, we consider Hénon maps $f(x, y) = (x^2 - a - by, x)$, where $(a, b) \in \mathbb{R} \times \mathbb{R}^\times$ is near the boundary of the horseshoe locus. We characterize their dynamics in terms of a symbolic dynamics. Concretely, we take a partition by two regions and consider the symbolic dynamics for this partition. Then, one can classify the behavior of Hénon maps according to the number of points which have a certain coding. Our result is an extension to any $b \neq 0$ of the previous one of Bedford and Smillie only for $b > 0$ close to zero. This is a joint work with Yutaka Ishii.

Zin Arai (Chubu)

On the disconnectedness of the Julia set for the Hénon map

In this talk, we propose a rigorous algorithm for proving the disconnectedness of the Julia set for the complex Hénon map. In the case of the one-dimensional quadratic map, it is well known that the Julia set is connected if and only if the orbit of the critical point of the map is bounded. In our case of the Henon map, since the map has no critical point, we need to keep track of a critical point of the Green function associated with the map. For this purpose, we develop a topological method that uses the plurisubharmonic nature of the Green function. This is a joint work with Yutaka Ishii.

Hiroyuki Inou (Kyoto)

A hole of the support of the bifurcation measure for the biquadratic family

We numerically computed the bifurcation measure for the family of biquadratic polynomials. Through 4D rotation in VR visualization, we found a "hole" in the support. In this talk, we want to discuss how to check its existence by rigorous computation.

なお、この時期の京都の宿泊はたいへん混み合います。お早めにホテルの予約をお願い致します。

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